**Insert School Logo**

**Semester One Examination 2019**

**Question/Answer Booklet**

**MATHEMATICS SPECIALIST**

**UNIT 3**

**Section Two:**

**Calculator-assumed**

Student Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Teacher’s Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: ten minutes

Working time for paper: one hundred minutes

**Material required/recommended for this section**

**To be provided by the supervisor**

This Question/Answer booklet

Formula Sheet (retained from Section One)

**To be provided by the candidate**

Standard items: pens(blue/black preferred), pencils(including coloured), sharpener, correction tape/fluid, erasers, ruler, highlighters

Special Items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations.

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Number of questions available | Number of questions to be attempted | Suggested working time (minutes) | Marks available | Weighting |
| Section One  Calculator—free | 6 | 6 | 50 minutes | 53 | 35% |
| **Section Two**  **Calculator—assumed** | **10** | **10** | **100 minutes** | **97** | **65%** |
|  | | | | 150 | 100% |

**Instructions to candidates**

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2019.* Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Section Two: Write answers in this Question/Answer Booklet. Answer **all** questions.

**Show all your working clearly.** Your working should be in sufficient detail to allow your

answers to be checked readily and for marks to be awarded for reasoning. Incorrect

answers given without supporting reasoning cannot be allocated any marks. For any

question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

1. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
2. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

* Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
* Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

1. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

# Section Two: Calculator–assumed 97 marks

This section has **ten (10)** questions. Attempt **all** questions.

Write your answers in the spaces provided.

Working time: 100 minutes

**Question 7 (4 marks)**

The complex number is such that . Use this information to evaluate the expression below.

(4 marks)

**Question 8 (10 marks)**

(a) Points A, B and C have position vectors and respectively.

Determine the value(s) of x and y so that:

(i) is a unit vector. (2 marks)

(ii) B divides the line segment AC in the ratio 2 : 3. (4 marks)

**(Question 8 – Continued)**

(b) Triangle ABC has its vertices at , and

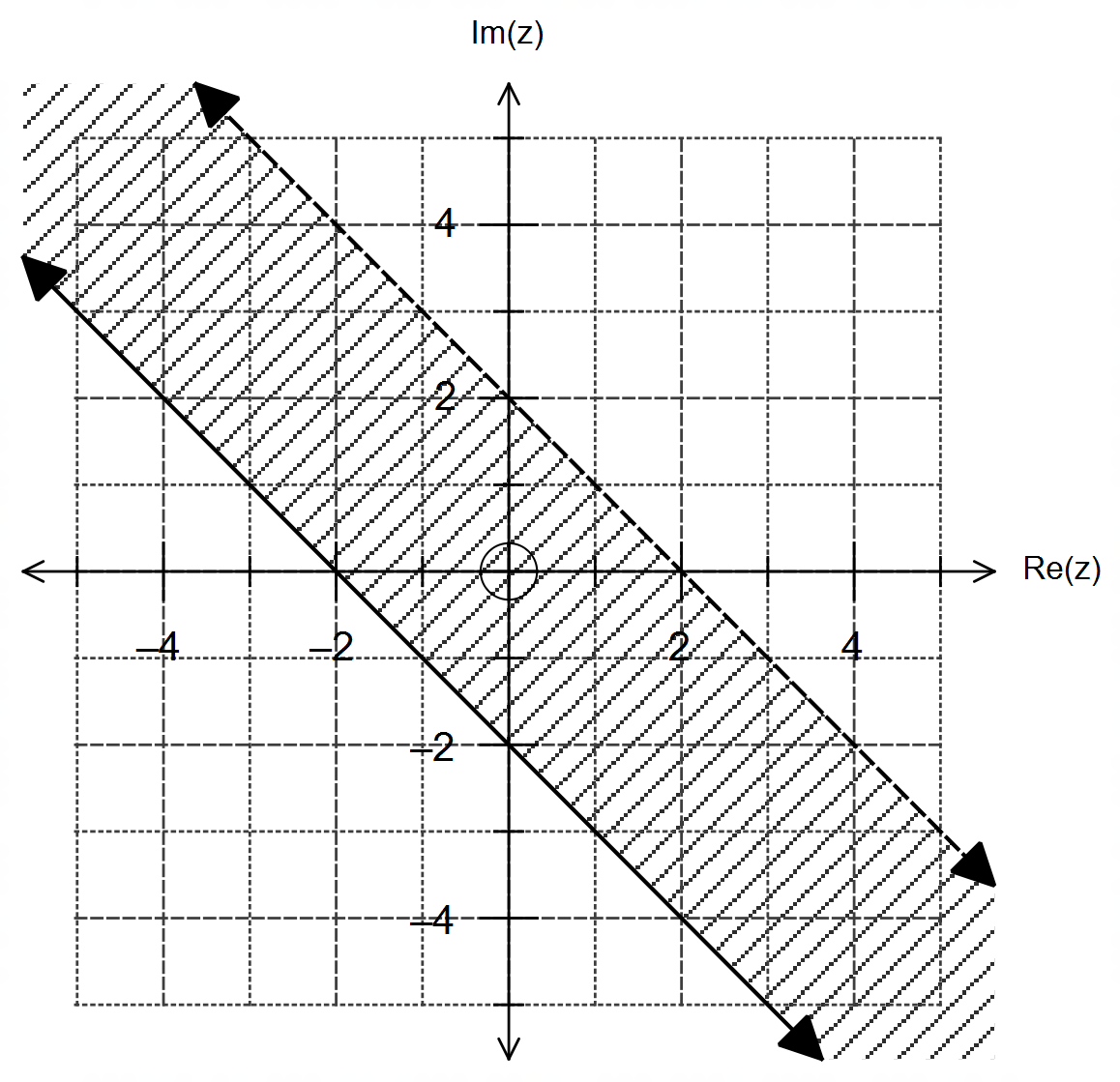
, where . Determine the value(s) of so that ΔABC is equilateral.

(4 marks)

**Question 9 (10 marks)**

(a) State the conditions on the complex number that describe the region given below.

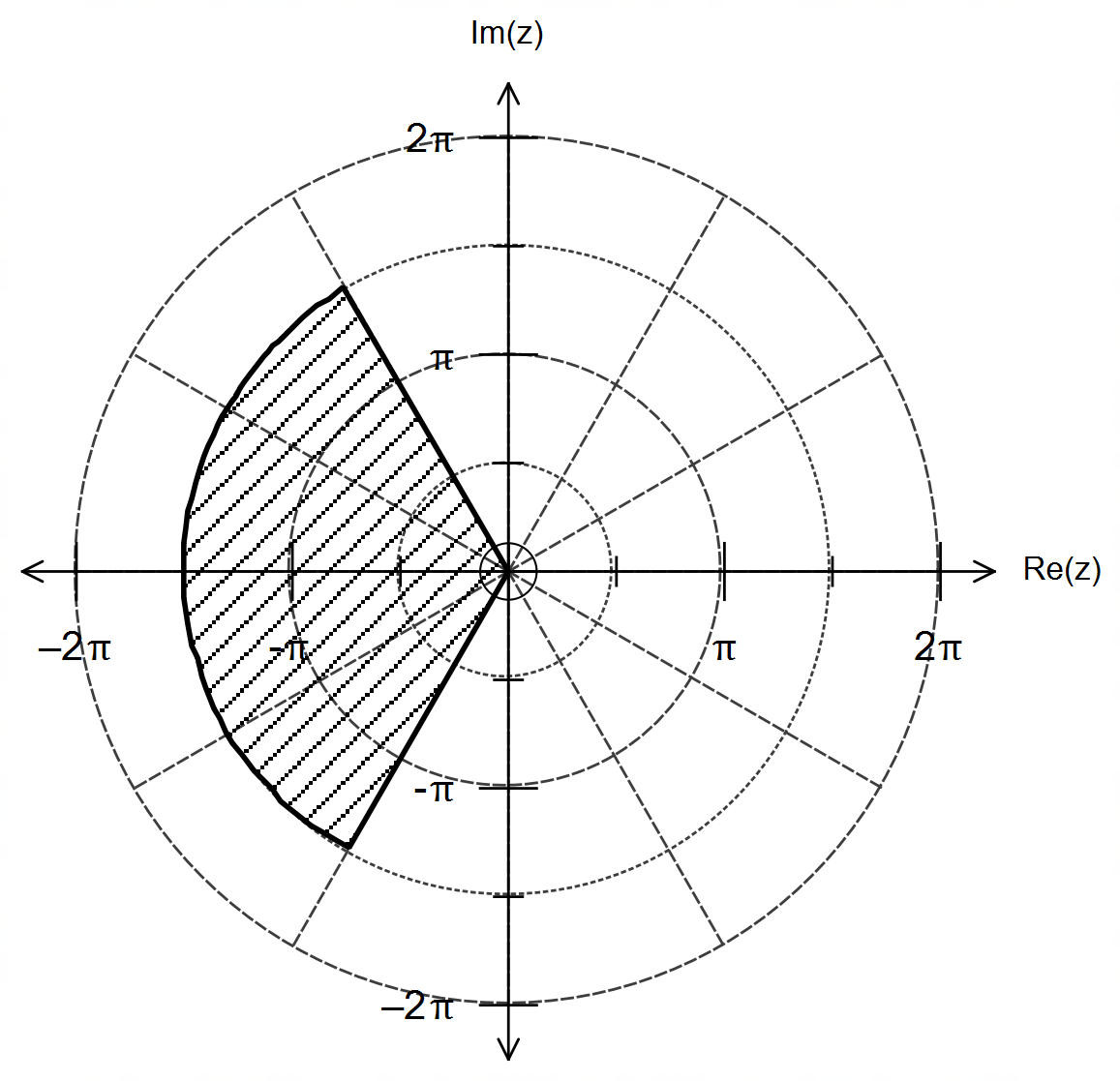
(3 marks)



(b) State the conditions on the complex number that describe the region given below,

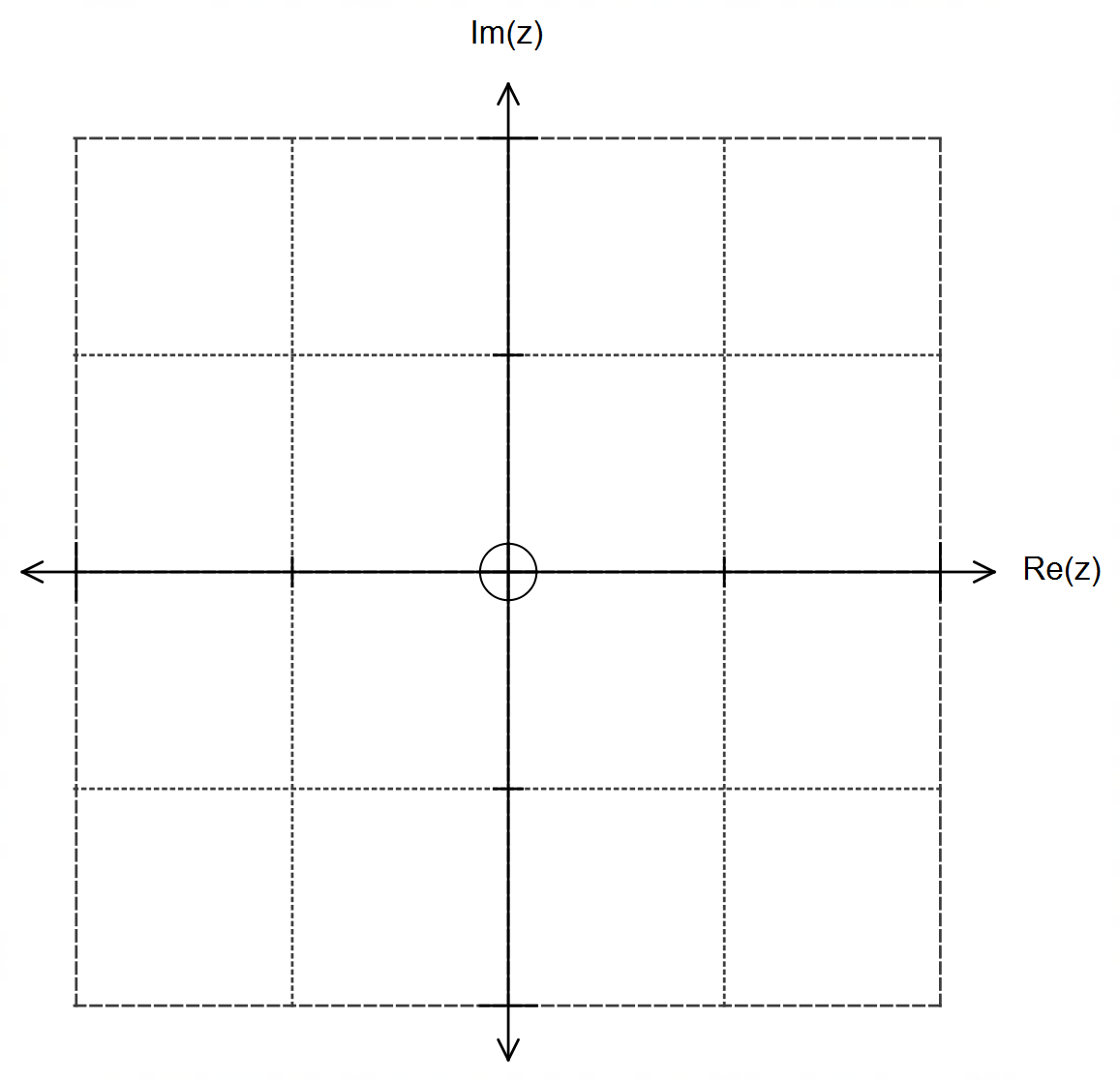
where and .

(3 marks)

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**(Question 9 – Continued)**

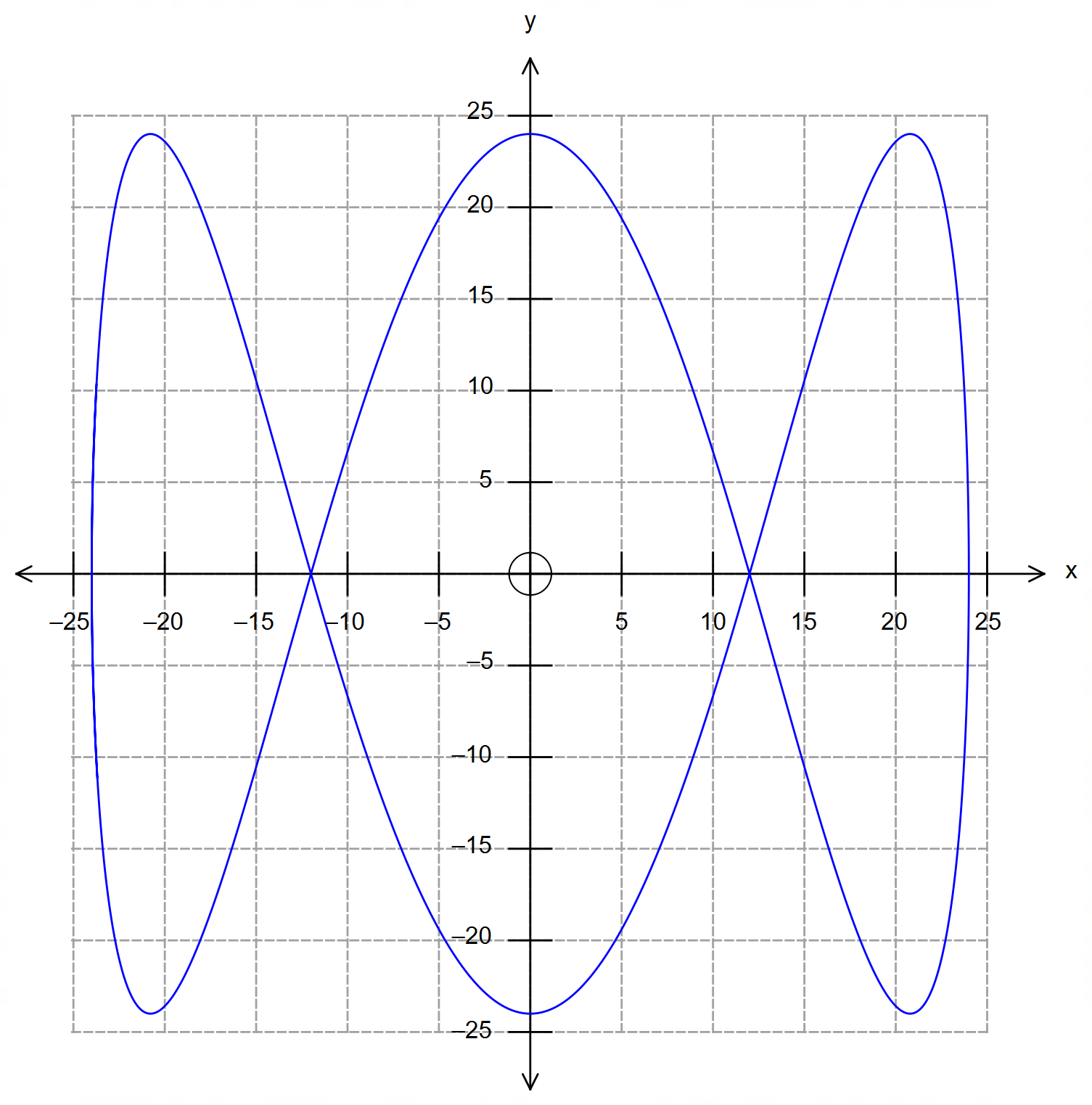
(c) Sketch the following set of complex numbers in the Argand plane that satisfy the condition:



Justify your answer with your working on the space below. (4 marks)

**Question 10 (17 marks)**

The path traced by the needle of an industrial welding machine is shown on the diagram below.



The position of the needle at any time is given by:

where is in cm, in seconds, is a unit vector pointing East and is a unit vector pointing North.

(a) Find the velocity of the needle at time . (2 marks)

**(Question 10 – Continued)**

(b) Determine the position and velocity vectors of the needle at times and , and

draw these on the diagram given. Label your answers clearly. (6 marks)

(c) How long does it take for the needle to complete one full cycle? (1 mark)

(d) Find the acceleration of the needle at time . (2 marks)

**(Question 10 – Continued)**

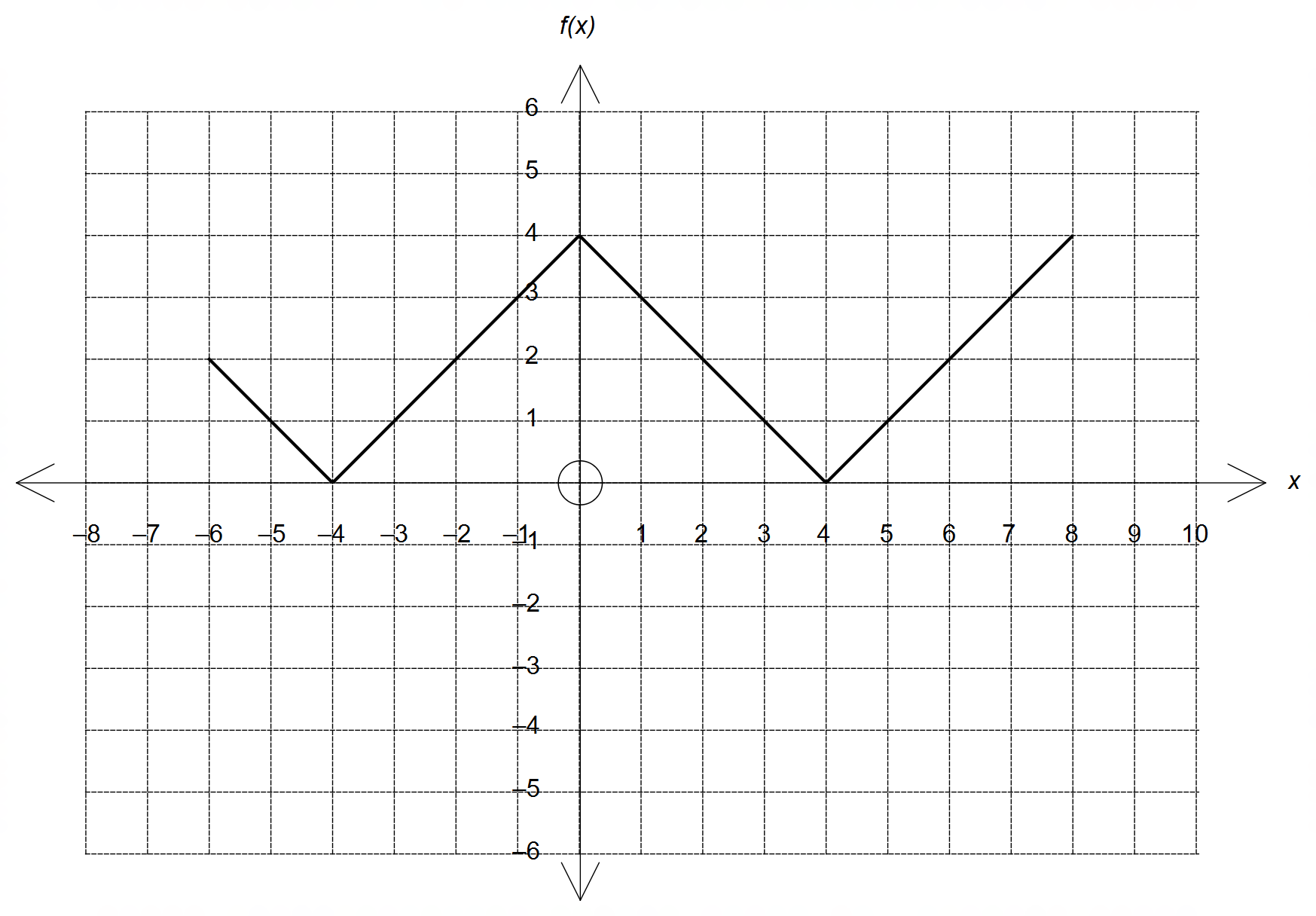
(e) Determine the speed of the needle as a function of . (2 marks)

(f) Use your calculator to determine the maximum speed of the needle and the location

where this first occurs. Clearly indicate this on the same graph. (4 marks)

**Question 11 (8 marks)**

The grid below shows the graph of for the domain .



(a) State the equation of . (2 marks)

(b) The function intersects for .

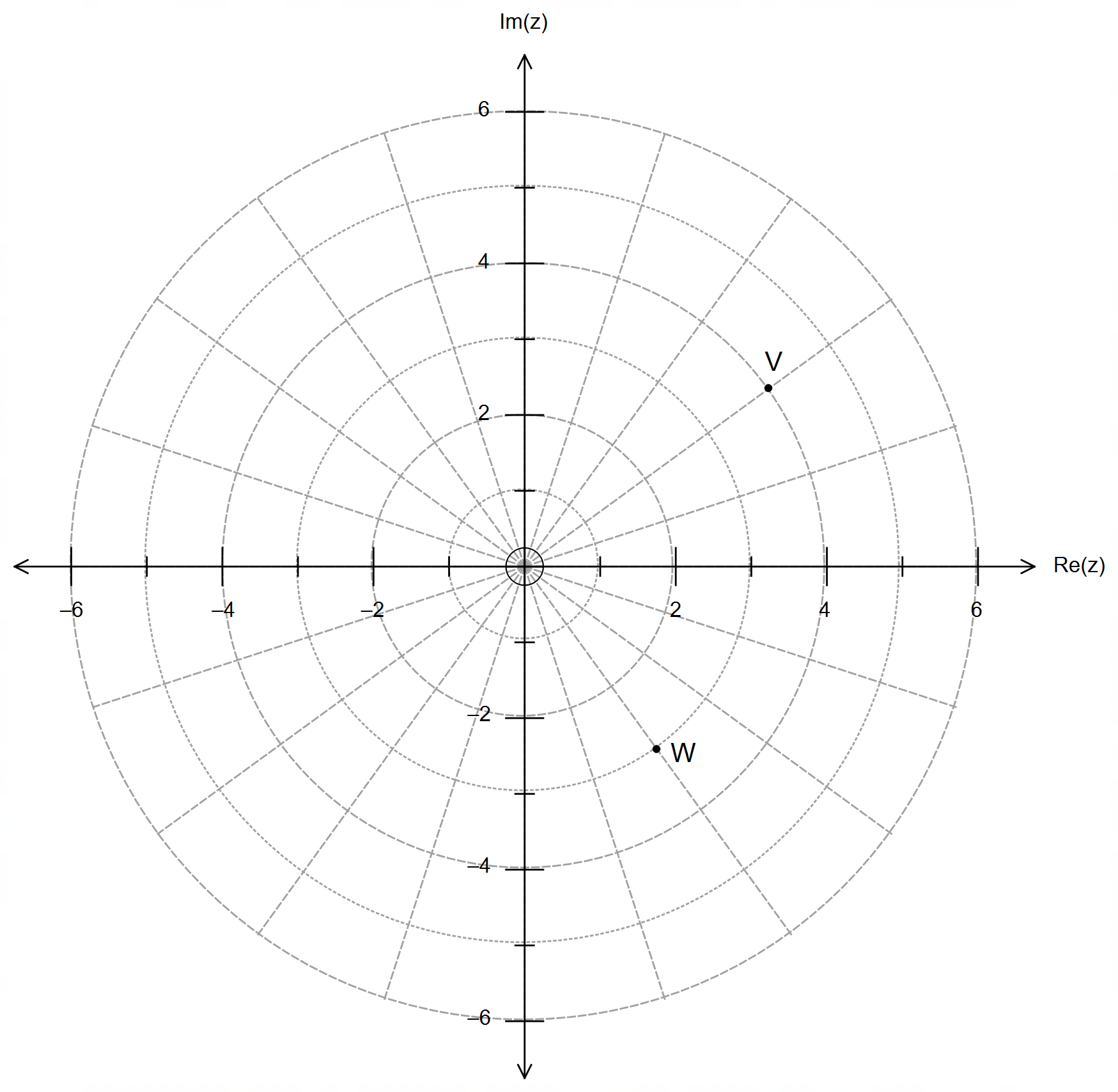
Determine the value of the constants and . (2 marks)

(c) State the independent conditions on the positive constants and so that the graph

of intersects the graph of exactly four times. (4 marks)

**Question 12 (12 marks)**

(a) The complex numbers and are shown on the polar grid below.



On the same grid above, mark the position of each of the following:

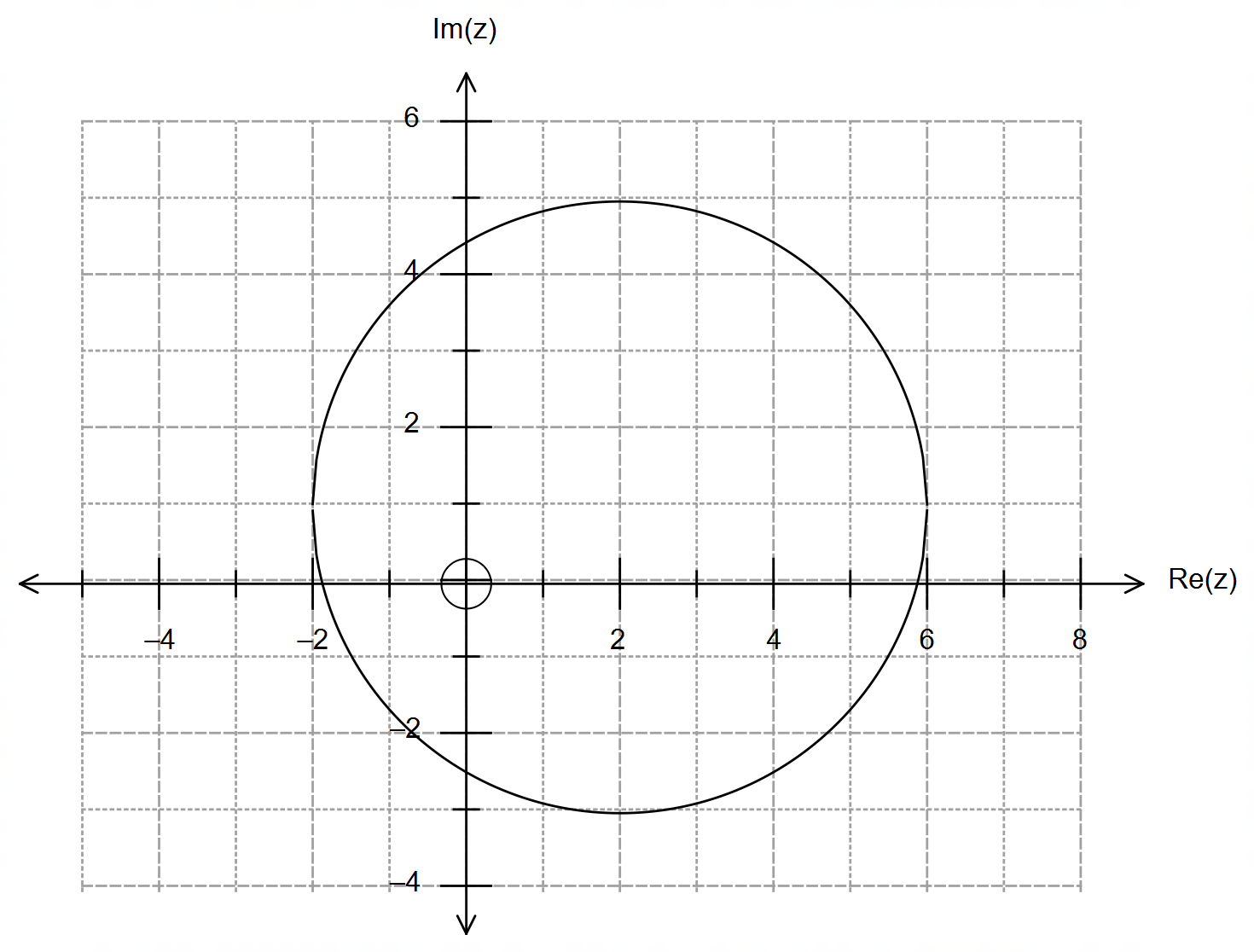
(i) (2 marks)

(ii) (2 marks)

(iii) (2 marks)

**Question 12 – Continued**

(b) The diagram below shows the region represented by



(i) Determine the minimum and maximum value of . (2 marks)

(ii) Determine the value(s) of for when . (4 marks)

**Question 13 (12 marks)**

A weather balloon is launched at am with a constant velocity of metres per second, where and are unit vectors east, north and up respectively.

Fifteen minutes later, at am, ground computers detect a malfunction in the weather balloon and they launch instantaneously an intercepting rocket to try and destroy the weather balloon.

The rocket is launched from the position metres relative to where the weather balloon was launched.

(a) Determine the speed of the weather balloon and the angle of its trajectory relative to level ground. Show working to justify your answers. (3 marks)

(b) Calculate the distance from the rocket to the weather balloon at am. (3 marks)

**(Question 13 – Continued)**

(c) The rocket is programmed to collide and destroy the weather balloon at am.

Determine the velocity of the rocket, and the height from level ground where the

collision occurs. (4 marks)

(d) If the speed of sound is m/s, determine the time elapsed for an observer at O,

the launching point of the weather balloon, to hear the collision after observing it. (2 marks)

**Question 14 (9 marks)**

Consider the complex number .

(a) Rationalise to show that . (2 marks)

(b) It can be shown that and .

Use this information to show that and . (2 marks)

**(Question 14 – Continued)**

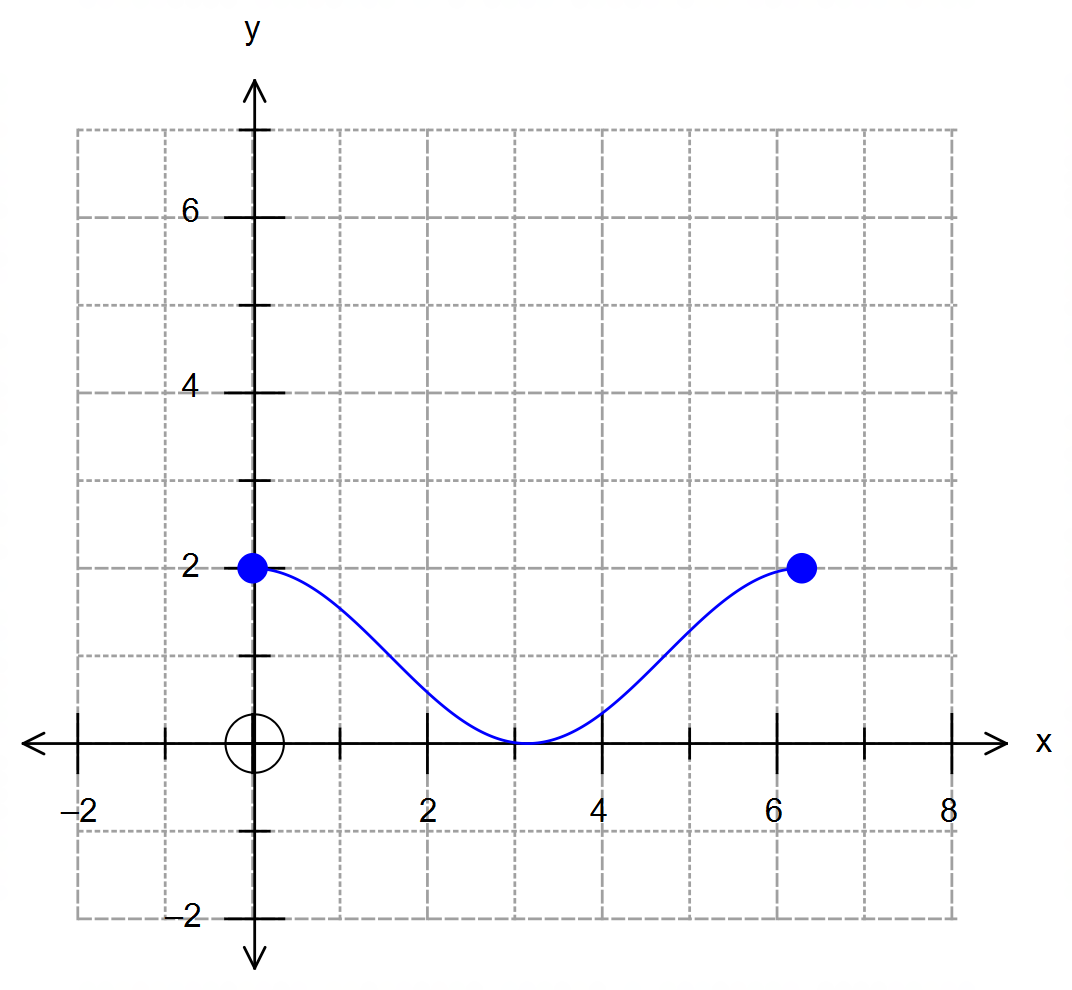
(c) Using your answers in (b), write an expression for and in terms of . (2 marks)

(d) Using your answers in (c), determine an expression for in terms of

and . Show working for full marks. (3 marks)

**Question 15 (10 marks)**

The graph below shows the function for the domain .



(a) Explain why the inverse function does not exist. (1 mark)

(b) The domain of can be restricted to so that exists.

(i) Determine the value of and then obtain an expression for . (3 marks)

(ii) State the domain and range of . (2 marks)

**Question 15 – Continued**

(c) Sketch on the same grid the function found in (b) (2 marks)

(d) The function is such that . Determine . (2 marks)

**Question 16 (5 marks)**

A particle moves along a path described by the equation , where .

Determine the Cartesian equation of the path described by the particle. Show working for full marks.

(5 marks)

**END OF QUESTIONS**

**Additional working space**

Question number(s): ……………………

**Additional working space**

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